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Inverse Domination Number of Euler Totient Cayley Graphs

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Abstract: G= (V, E) be a simple graph. D is a subset of V is said to be a dominating set if every vertex in V-D is adjacent to at least one vertex in D. The minimum cardinality of a dominating set is called domination number and it is denoted by γ. In this paper we investigate the inverse domination number of Euler Totient Cayley graph.

Key Wards: Domination number, Dominating set, Euler Totient Cayley Graph, Inverse Dominating Number, Inverse Domination Set.

1. INTRODUCTION:

The origin of graph theory is supposed to be traced from the study of Konigsberg bridge problem during 1736by the great mathematician Leonard Euler. That attracted many mathematicians to contribute much more in the study of graph theory. The study of domination in graphs originates around 1850 with the problem of placing minimum number of queens on an nxn chess board. So as to cover or dominate every square. The theory of domination in graphs introduced by O. Ore and C. Berge is an emerging area of research in graph theory today. C. Berge presents the problem of five queens, namely, place five queens on the chess boar so that every square is covered by at least one queen.

Kulli V.R. et al [7] introduced the concept of Inverse domination in graphs. Let G= (V, E) be a simple, finite, undirected, connected graph. Any undefined term in this paper may be found in Haynes T. W et al (1998). A nonempty set D V of a graph G is a **dominating set** of G if every vertex in V-D is adjacent to some vertex in D.

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The domination number (G) is the minimum cardinality taken over all the minimal dominating sets of G. Let D be the minimum dominating set of G. If V-D contains a dominating set D then D is called the **Inverse dominating set** of G w.r.to D. The **Inverse dominating number** (G) is the minimum cardinality taken over all the minimal inverse dominating sets of G.

Throughout in this work G= (V, E) be a simple graph.

2. Preliminaries:

Cayley Graph 2.1:

Let $(X, .)$ be a group and S, a symmetric subset of X not containing the identity element e of X. The graph G whose vertex set $V = X$ and edge set E = $\{(g, gs) / s \in S\}$ is called the Cayley graph of X corresponding to the set S and it is denoted by $G(X, S)$.

Euler Totient Cayley Graph 2.2:

For each positive integer, let Zn be the additive group of integers modulo n and S be the set of all numbers less than n and relatively prime to n. The Euler totient Cayley graph G(Z , φ) is defined as the graph whose vertex set V is given by $Zn = \{0,1,2, \ldots . . n-1\}$ and the edge set is given by $E = \{(x, y)/ x - y \in S$ or $y - x \in S\}$.

3. Main Results:

Theorem 3.1: If n is a prime, then the inverse domination number of G $(Z_{n,}\varphi)$ is 1

Proof: let n be a prime. Then G $(Z_{n,}\varphi)$ is a complete graph. Let D=$\{u\}$, where u is any vertex in V. Then every vertex t

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in V is adjacent to vertex u. Thus every vertex in V-D is adjacent to u. so that D forms a dominating set in G (Z_{n},φ). Since $|D|$=1, it is evident that D is a minimum dominating set in G (Z_{n},φ).

Now, V' =V-D is a vertex set and D' ={v}, where v is any vertex in V' . Then every vertex in V' -D' is adjacent to vertex v inD'. So D' forms a dominating set.

∴ D' is a dominating set with respect to D. $|D'|$=1. $D'$is a minimum inverse dominating set in G (Z_{n},φ).

∴ Inverse domination number of G (Z_{n},φ) is 1, when n is a prime. ∴ $\gamma'(G(Z_n,\ \emptyset))$=1

Theorem 3.2: If n is a power of a prime, then the inverse domination number of G (Z_{n},φ) is 2.

Proof: Consider G (Z_{n},φ) for $n = p^{\alpha}$, where p is a prime. The vertex set V of G (Z_{n},φ) is given by V={0,1,2,3, … …. . $p^{\alpha} - 1$}. V can be decomposed in to the following disjoint subsets.

1. The set S of integers relatively prime to n
2. The set M of multiples of P
3. Singleton set{0}.

If $v \epsilon S$, vertex v is adjacent to the vertex 0. Thus a single vertex 0 dominates all the vertices of S. Now, consider the vertices of M. These vertices are multiples of p. that is $v \epsilon M$ then

$v = rp$,where $r < p^{\alpha-1}$. Then for any vertex $t \epsilon S$, $v - t \notin M$ and $v - t \neq 0$. So that $v - t \in S$. This implies that v and t are adjacent. Thus a single vertex t in S dominates all the vertices of M.

Let $D = \{0, t\}$. Then as per above discussion D becomes a dominating set of G (Z_{n},φ) with cardinality 2. So a single vertex cannot dominate the remaining p^{α}-1 vertex. So we require at least two vertices.

∴ D is a minimum dominating set of G (Z_{n},φ)

∴ $\gamma($G (Z_{n},ϕ)) =2.

Now, V' is a vertex set deleting a dominating set D from the vertex set V

V'=V-D={0,1,2,3 … … . . $p^{\alpha} - 1\} - \{0, t\}$

Consider the vertex $v \in M$.Then $v = rp$, where$r < p^{\alpha-1}$. Then for any vertex $x \in S$ other than the element in D, $x - v\ (or)\ v - x\ \epsilon\ S$. So that x and v are adjacent. x dominates all the vertices of M but not all the vertices in S.

Consider the vertex $x \epsilon\ S$ other than the element in D. Then for any vertex $y \epsilon\ M$,

$x - y\ (or) y - x\ \epsilon\ S$. So y and x are adjacent. y dominates all the vertices in S.

∴ D'= $\{(x, y) / x \in S, x \notin D, y \in M\}$.

D' Dominating all the vertices in V'.

D' forms a minimum dominating set with respect to D with minimum cardinality 2.

∴ $\gamma'(G(Z_n,\ \emptyset))$=2.

Theorem 3.3: The inverse domination number of G (Z_{n},φ) is 2, if n=2p where p is an odd prime.

Proof: Let us consider G (Z_{n},φ) for n=2p, p is an odd prime. Then the vertex set V={0,1,2,3, … … . $p - 1$}. Let V can be decomposed in to the following disjoint subsets.

1. The set S of odd integers which are less than n and relatively prime to n
2. The set M of non-zero even integers.
3. The set D of integers 0 and p.

We now show that D= {0, p} is a dominating set of G (Z_{n},φ). By the definition of edges in G (Z_{n},φ) it is clear that the vertices in S are adjacent to 0 and hence they are dominated by the vertex 0.

Now consider the elements of M which are non-zero even numbers. If $v \in M$then $v - p$ is an odd number because p is an odd prime. Hence $v - p\ (or) p - v \in S$.That is vertex$v \in M$ is adjacent to vertex p. This implies that the vertices of M are dominated by vertex p. Thus every vertex in $V - D$ is

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dominated by the vertices of D. Therefore D is a dominating set of G (Z_n,φ).

∴D= {0, P} is the minimum dominating set with cardinality 2.

Thus∴ $\gamma($ G (Z_n,ϕ)) =2.

V'=V-D={$1,2,3, \ldots \ldots .., p-1, p-2, \ldots \ldots ., 2p-1$}.

Now consider the elements of S which are relatively prime to n. If $s \in S, v \in M$ where M is the set non-zero even integers. By the definition of edges s is adjacent to the all the vertices of M except the vertex u where $u=(p+s)$. That is $v-s\ (or) s-v \in S$ But $u-s=(p+s)-s=p \notin S$.

$\therefore v-s\ (or) s-v \in S$ but $u-s\ (or) s-u \notin S$.

Thus s is adjacent to all the vertices of M except u.

Now, consider the vertex $u=(p+s) \in M$ where M is the set of non-zero even integers. If $u \in M, t \in S$ then $u-t\ (or) t-u \in S$ But $u-s\ (or) s-u \notin S$.

The vertex u is adjacent to the all the vertices of S except s.

$\therefore D'=\{s,u\}$ is a dominating set.

The resulting graph is (p-2) regular graph. That is every vertex of resulting graph is of degree p-2. Hence a single vertex cannot dominate the rest of 2p-3 vertices. Therefore $D'=\{s,u\}$ becomes a minimum dominating set with cardinality 2.

$\therefore \gamma'(G(Z_n,\ \emptyset))$=2.

Theorem 3.4: Suppose n is neither prime nor 2p. Let $n={p_1}^{\alpha_1}, {p_2}^{\alpha_2}, \ldots \ldots .., {p_k}^{\alpha_k}$, where $p_1, p_2, \ldots, p_k$ are primes and $\alpha_1, \alpha_2, \ldots, \alpha_k$ are integers≥1. Then the inverse domination number of G (Z_n,φ) is λ+1, where λ is the length of the longest stretch of consecutive integers in V, each of which shares a prime factor with n.

Proof: Suppose $n={p_1}^{\alpha_1}, {p_2}^{\alpha_2}, \ldots \ldots .., {p_k}^{\alpha_k}$, where $p_1, p_2, \ldots, p_k$ primes and $\alpha_1, \alpha_2, \ldots, \alpha_k$ are integers≥1 and n is neither prime nor2p. Let the vertex set V of G (Z_n,φ) is V= {0, 1, 2…, n-1}. V can be decomposed in to the following disjoint subsets.

1 The set S of integers relatively prime to n

2 The set X= {Sᵢ}, where Sᵢ is a collection of consecutive integers in V such that for every x in Sᵢ, GCD (x,n)>1.

3 The singleton set {0}.

Uma Maheswari [3] proved D= {0, 1, 2, … … . , λ} is minimum dominating set .

Therefore the domination number of $G(Z_n,\ \emptyset)$ is λ+1.

$V'=\{\lambda+1, \lambda+2, \lambda+3, \ldots \ldots, n-1\}$ is a vertex set after deleting the dominating set from the vertex set V.

The degree of induced sub graph of G is equal to the degree of the graph G.

Hence $D'=\{\lambda+1, \lambda+2, \lambda+3, \ldots \ldots, 2\lambda+1\}$ is a minimum dominating set with respect to D.

$\therefore D'$ is a inverse dominating set of $G(Z_n,\ \emptyset)$ with cardinality λ+1.

$\therefore \gamma'(G(Z_n,\ \emptyset))$= λ+1.

4. Conclusion: I observe that the domination number and inverse domination number of $G(Z_n,\ \emptyset)$ is equal.

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